**Delay Discounting as a Latent Variable**

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Delay discounting refers to the tendency for individuals to prioritize immediate rewards over those that are delayed or set in the future (Madden et al., 2023). Individuals who exhibit high delay discounting, meaning they heavily discount or devalue delayed rewards in favor of immediate gratification, are often prone to impulsive decision-making and maladaptive behaviors such as substance abuse, gambling, credit card debt, and poor academic performance (Wölfling et al., 2020).

While prior research has consistently linked high delay discounting to a variety of significant behavioral outcomes, the relationships identified across studies are varied and, at times, inconsistent (Bickel et al., 2001; Duckworth & Seligman, 2005; Kirby et al., 1999). One potential explanation for this inconsistency is the variation in how delay discounting is operationalized across studies. Traditionally, delay discounting has been measured using the Kirby Delay Discounting Task (DDT), a monetary choice questionnaire that presents participants with 27 hypothetical choices between smaller-sooner and larger-later rewards (Kirby & Marakovic, 1996). However, alternative approaches, such as titration tasks, matching tasks, and experiential discounting tasks, have also been employed. Additionally, studies differ in their use of monetary versus non-monetary rewards and hypothetical versus non-hypothetical rewards.

Despite the proliferation of methods used to measure delay discounting, there is limited research exploring the latent factors that may underlie the construct. Understanding these latent factors is critical for unifying the measurement approaches and providing greater clarity on the relationships between delay discounting and associated behavioral outcomes. To address this gap, the current study seeks to examine the underlying latent factor structure of delay discounting using confirmatory factor analysis (CFA). The study further aims to evaluate how these latent factors relate to previously cited behavioral outcomes of interest.

The specific hypotheses and research question guiding this study are as follows:

***Hypothesis 1:*** The delay discounting items will share enough common variance for a latent variable CFA to fit the data well.

***Hypothesis 2:*** Delay discounting will exhibit stronger correlations with associated constructs at the latent factor level compared to the observed level (e.g., scale scores).

***Research Question 1:*** Will a one-factor or multi-factor model provide the best fit to the data?

This project specifically focuses on examining the latent factor structure underlying delay discounting and, therefore, addresses only Hypothesis 1 and Research Question 1.

**Method**

**Study Design and Participants**

This was an observational study. Thus, it did not involve experimental manipulations or group assignments. Participants were recruited from the undergraduate SONA participant pool at Montclair State University (MSU), resulting in a sample of 500 students.

**Procedure**

Data were collected using the Qualtrics survey platform. Participants first completed 12 delay-discounting items, followed by a series of self-reported scales measuring related constructs, and concluded with demographic questions. The delay-discounting items were structured as matching tasks in which participants indicated the value of a hypothetical monetary reward that would make them indifferent to another specified reward. For example, participants might decide between receiving $40 immediately or $\_\_\_ in one month. Six of the 12 items required delaying an immediate reward to a future date, as the example, while the remaining six involved expediting a future reward, such as choosing between receiving $\_\_\_ now or $1500 in 12 months.

The self-reported measures included the Academic Procrastination Scale – Short Form (Yockey, 2016), consisting of five items assessing tendencies to delay academic tasks. The Goal Setting Formative Questionnaire (Erickson, Soukup, Noonan, & McGurn, 2022) included 19 items evaluating goal-setting behaviors and attitudes. The Barratt Impulsiveness Scale – Revised (Barratt, 1995) contained 30 items measuring impulsivity across multiple dimensions. The Alcohol Use Disorders Identification Test (AUDIT; World Health Organization, 2001) included 10 items to screen for harmful patterns of alcohol consumption. The Discounting Inventory – Delay Items (Malesza & Ostaszewski, 2020) contained 12 items designed to further assess delay-discounting tendencies. Finally, demographic information, including age, ethnicity, race, and gender, was collected.

**Data Analytic Plan**

The data for this study were analyzed using R (R Core Team, 2023, Version 4.3.0). Several packages were utilized for the analysis, including tidyverse, psych, lavaan, haven, semTools, and stringr. After importing the dataset into the R environment, an initial check revealed that 3 out of the 500 responses were unfinished. While further verification indicated that all data for the discounting task columns required for the CFA were complete, a closer inspection of the dataset revealed that many values were effectively missing, although they were not marked as missing (NAs). Additionally, the discounting task responses were stored as character types instead of numeric values. These issues necessitated a series of data-cleaning steps, as outlined below.

***Data Cleaning***

A total of nine responses included a ‘$’ symbol before their entries, which caused the column to be interpreted as character data. To resolve this, the ‘$’ symbol was removed from these responses. Additionally, some responses contained commas (e.g., in monetary values), which were also removed to ensure proper numeric formatting.

In a few cases, a period appeared at the end of the response instead of serving as a decimal point. If a period was found at the end of a response, it was removed, as leaving it in place would result in valid responses being coerced to NAs during conversion to numeric format. Furthermore, five participants included additional text in their responses, which could not be reliably converted for mathematical purposes; these entries were converted to NAs. Finally, any empty cells in the dataset were also converted to NAs.

***Missing Data Analysis***

To ensure proper handling of missing data during the CFA, a missing value analysis was conducted to determine whether the missing data were random. A new variable was created to differentiate between complete cases (Complete = 1) and incomplete cases (Complete = 0). Bivariate tests such as Independent samples t-tests and chi-square tests were then performed to assess whether there were significant differences in any demographic variables between the complete (n = 489) and incomplete cases (n = 11).

***Discounting Rates Conversion***

The monetary responses on each discounting trial were converted to discounting rates using the following formula:

r =

X­t is the magnitude of the smaller-sooner reward, Xt+h is the magnitude of the larger later reward and h is the additional delay associated with the larger later reward.

***Outlier Detection***

Outliers were identified by converting discount rates to z-scores. Discount rates with z-scores greater than +3 or less than -3 were flagged as outliers and converted to NAs. Additionally, all negative discount rates were converted to missing values (NAs). Negative discount rates are conceptually inconsistent with psychological and economic principles, often indicating that a participant either misunderstood the task or responded carelessly.

***Multivariate Analysis***

Before conducting the CFA, assumptions were thoroughly tested. The sample size remained sufficient for analysis (n > 200) even after removing outliers. As multivariate outliers are typically not an issue once univariate outliers have been addressed, no further tests for multivariate outliers were performed. Other assumption checks included an evaluation of multicollinearity among variables and the assessment of linearity between the latent factors and observed variables.

To address Hypothesis 1 and Research Question 1, three Confirmatory Factor Analysis (CFA) models were developed using the lavaan package in Rstudio. Missing data were imputed using the Full Information Maximum Likelihood (FIML) method, as the data were determined to be missing at random.

The first model was a single-factor model, in which all individual discount rate items were loaded onto one latent factor. The second model was a two-factor model: items involving the delay of an immediate reward were loaded onto a latent factor labeled "defer," while items related to expediting a future reward were loaded onto a separate factor labeled "expedite." The third model was a four-factor model, which further distinguished between the magnitude of rewards. Specifically, for both defer and expedite items, three items involved smaller rewards (less than $100), and three involved larger rewards (greater than $800). This model included the latent factors "defer\_small," "defer\_large," "expedite\_small," and "expedite\_large."

Model fit was assessed using the Comparative Fit Index (CFI), Root Mean Square Error of Approximation (RMSEA), and Standardized Root Mean Square Residual (SRMR). While chi-square values were reported, they were not used for model evaluation because large sample sizes typically result in significant chi-square values, irrespective of the model’s quality. Finally, nested model comparisons were conducted using likelihood ratio tests to evaluate relative model fit.

**Results**

**Preliminary Analyses Findings**

The initial data-cleaning processes (removal of dollar signs, commas, text responses, and empty cells) collectively resulted in a total of 27 missing values (NAs) across the dataset. All the missing values resulted from 11 data points/participant responses. The descriptive statistics of the relevant variables are provided in Table 1.

Following the initial cleaning, missing value analysis was conducted to determine whether the missing data were random. Independent samples t-tests showed no significant difference in age between complete cases (*M* = 20.14, *SD* = 3.41) and incomplete cases (*M* = 20.00, *SD* = 2.23), *t* (6.41) = -0.17, *p* = 0.87. Similarly, chi-square tests found no significant associations between case completeness and gender (*χ*² (4, *N* = 500) = 0.65, *p* = 0.95) or race (*χ²* (6, N = 500) = 5.81, *p* = 0.44). These results suggest that the missing data were not systematically related to the demographic variables examined.

Following the missing data analysis, monetary responses were converted to discount rates using the formula described earlier. Outliers were identified based on z-scores and converted to NAs. Negative discounting rates, which were also considered outliers, contributed to a total of 593 missing values, roughly corresponding to 41 responses out of 500. Despite this, sufficient data remained for conducting the CFA.

A test for multicollinearity revealed that three items—"expedite3," "expedite4," and "expedite6"—exhibited near-perfect correlations, indicating problematic multicollinearity (see Table 2). Linearity assumptions between the observed variables and latent factors were evaluated after developing specific models and calculating factor scores. All three models were found to meet the assumptions of linearity.

**Multivariate Analyses Findings**

The sample size for all three models was 497, following the use of FIML method for missing data imputation. Our first model was unidimensional, where each of the 12 items loaded onto a single factor called “discounting,” representing overall discount rates (see Figure 1). The Chi-square index was statistically significant (*χ2*(54) =1589.43, *p*<.001), indicating a likely misfit. The CFI value of .72 indicated poor fit. The RMSEA = .24 (90% CI [.23, .25]) also suggests serious problems with the one-factor model structure. The SRMR value of .109 was above the cutoff value of .10 for a good fit. The Cronbach’s alpha, omega construct reliabilities, and average variance extracted values are all within acceptable ranges, with the latent factor accounting for 60% of the total variance in the sample. However, the fit indices indicate a poor model fit, failing to provide support for Hypothesis 1.

The second model had two factors, where half the items were loaded onto the factor “defer,” and the other half were loaded onto the factor “expedite” (see Figure 2). The Chi-square index was statistically significant (*χ2*(53) = 408.79, *p*<.001). The CFI value of .94 suggests an acceptable fit. The RMSEA = .12 (90% CI [.11, .13]) was higher than the recommended .10 suggesting a marginally poor fit of the model. The SRMR value of .04 suggests an excellent model fit. Taken together, these fit indices indicate that the two-factor model provides an acceptable fit to the data. Additionally, Cronbach’s alpha, omega construct reliabilities, and average variance extracted values are all within acceptable ranges, with the first latent factor “defer” accounting for 68% of the total variance and the second latent factor “expedite” accounting for 69% of the total variance in their loaded items.

The third model was a four-factor model that had the following factors “defer\_small”, “defer\_large”, “expedite\_small”, and “expedite\_large” and each factor had three items loading onto them (see Figure 3). The Chi-square index was statistically significant (*χ2*(48) = 263.24, *p*<.001). However, as mentioned previously, large sample sizes typically result in significant chi-square values, irrespective of the model’s quality. The CFI value of .96 suggests an excellent fit to the data as it exceeds the recommended value of .95. The RMSEA = .095 (90% CI [.084, .106]) is lower than the recommended .10 suggesting an acceptable fit to the model. The SRMR value of .035 suggests an excellent model fit. Taken together, these fit indices indicate that the four-factor model provides an excellent fit to the data. Additionally, omega construct reliabilities and average variance extracted (AVE) values fall within acceptable ranges, with the first latent factor, "defer\_small," accounting for 65% of the total variance, the second latent factor, "defer\_large," explaining 72%, the third latent factor, "expedite\_small," explaining 67%, and the fourth latent factor, "expedite\_large," explaining 70% of the variance in their respective loaded items. However, Cronbach’s alpha values were below the acceptable range, ranging from .47 to .53. This is likely due to only three items being loaded onto each latent factor.

The likelihood ratio test between the single-factor model and the two-factor model revealed a significant difference, *χ²(*1) = 1180.6, *p* < .001. This indicates that the two models are statistically different, with the two-factor model providing a better fit. The likelihood ratio test between the two-factor model and the four-factor model revealed a significant difference, *χ²(*5) = 145.55, *p* < .001. This indicates that the two models are statistically different, with the four-factor model providing even a better fit than the two-factor model (see Table 3).

**Discussion**

The results of the CFA analyses did not support Hypothesis 1, as the delay discounting items lacked sufficient common variance for a unidimensional latent variable model to fit the data effectively. The two-factor CFA model, which separated items into “defer” and “expedite” factors, demonstrated a significant improvement over the single-factor model. Fit indices and the likelihood ratio test indicated an acceptable model fit. However, the four-factor model, comprising "defer\_small," "defer\_large," "expedite\_small," and "expedite\_large," provided the best fit to the data. These findings suggest that a multifactor structure with four distinct factors is the most accurate representation of the data, offering a better recreation of the covariance matrix.

These results have several important implications. First, they highlight that the processes of delaying an immediate reward and expediting a future reward represent distinct latent factors within delay discounting. Moreover, the magnitude of the reward—whether small or large—further differentiates these factors, resulting in the observed four-factor model. This study makes a significant contribution to the delay discounting and decision-making literature by offering a nuanced understanding of the underlying constructs of delay discounting. Future research could explore how these latent factors relate to behaviors historically associated with delay discounting, potentially addressing inconsistencies in the literature and clarifying how construct operationalization contributes to these discrepancies.

The four-factor model also has practical implications for research and application. For example, it could inform strategies related to consumer decision-making and policy interventions. Interventions aimed at encouraging delayed gratification or expedited rewards might be more effective if tailored to specific factors, such as the magnitude and timing of the reward. Similarly, organizations could use these insights to refine reward structures or marketing strategies, aligning them with distinct reward-related behaviors.

**Limitations and Future Directions**

Despite its contributions, this study has several limitations. One notable issue was multicollinearity among three items, which could have been addressed by combining them or retaining only one. However, due to the limited number of items (12), removing two would have left fewer than three items in two of the four factors, compromising the model structure. Another limitation was the relatively low Cronbach’s alpha values for the four-factor model, suggesting limited internal consistency. This is likely attributable to the small number of items per factor (three items), which may have affected the reliability of the constructs. Future studies should include a larger number of items and experiment with models incorporating additional latent factors.

Additionally, this study did not examine potential covariates that could influence the relationships between the "defer" and "expedite" factors. Future research could explore how demographic variables, personality traits, or situational contexts interact with these latent factors to shape decision-making behavior. Researchers might also investigate alternative dimensions of discounting behavior or test configurations with more factors to further refine the model. Lastly, validation of the four-factor model across diverse populations and contexts would enhance its generalizability and robustness, ensuring its utility in various applications.

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scale. *Psychological Reports*, *118*(1), 171-179.

**Table 1**

*Descriptive Statistics for Relevant Variables*

|  |  |  |
| --- | --- | --- |
| Variables | *M* | *SD* |
| Discounting Items |  |  |
| Defer1 | 308.07 | 496.22 |
| Defer2 | 2104527.86 | 44809830.83 |
| Defer3 | 3573.86 | 21950.61 |
| Defer4 | 4250.79 | 7690.31 |
| Defer5 | 26356.55 | 63312.86 |
| Defer6 | 11060.97 | 87610.38 |
| Expedite1 | 525.42 | 10502.36 |
| Expedite2 | 2202.11 | 25966.80 |
| Expedite3 | 289.55 | 4519.78 |
| Expedite4 | 806.92 | 6799.95 |
| Expedite5 | 1574.30 | 12236.22 |
| Expedite6 | 2106.61 | 44815.01 |
| Age | 20.14 | 3.40 |
|  |  |  |
|  | *n* | % |
| Gender |  |  |
| Male | 119 | 23.99% |
| Female | 362 | 72.98% |
| Non-binary/gender fluid or gender queer | 12 | 2.42% |
| I am not sure or questioning | 0 | 0.00% |
| Other | 1 | 0.20% |
| Prefer not to say | 2 | 0.40% |
| Race |  |  |
| White | 253 | 51.11% |
| Black or African American | 85 | 17.17% |
| American Indian or Alaska Native | 2 | 0.40% |
| Asian | 38 | 7.68% |
| Native Hawaiian or Pacific Islander | 3 | 0.61% |
| Other | 67 | 13.54% |
| Prefer not to say | 47 | 9.49% |

*Note.* Items 1 to 6, labeled "defer," reflect the delay of an immediate reward, while items 7 to 12, labeled "expedite," pertain to the acceleration of a future reward.

**Table 2**

*Correlation Table of Delay Discounting Items*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1.defer1 |  |  |  |  |  |  |  |  |  |  |  |
| 2.defer2 | -.30 |  |  |  |  |  |  |  |  |  |  |
| 3.defer3 | .45 | -.23 |  |  |  |  |  |  |  |  |  |
| 4.defer4 | .86\*\* | -.32 | .46 |  |  |  |  |  |  |  |  |
| 5.defer5 | .86\*\* | -.28 | .50 | .96\*\* |  |  |  |  |  |  |  |
| 6.defer6 | -.50 | .75\*\* | -.39 | -.53 | -.47 |  |  |  |  |  |  |
| 7.expedite1 | -.22 | -.16 | -.19 | -.17 | -.25 | -.33 |  |  |  |  |  |
| 8.expedite2 | -.47 | .10 | -.39 | -.48 | -.49 | .06 | -.17 |  |  |  |  |
| 9.expedite3 | -.51 | -.15 | -.47 | -.53 | -.50 | .52 | -.24 | .17 |  |  |  |
| 10.expedite4 | -.53 | -.15 | -.48 | -.55 | -.52 | .51 | -.23 | .20 | 1.00\*\* |  |  |
| 11.expedite5 | -.63 | -.03 | -.55 | -.65\* | -.64\* | .35 | -.27 | .80\*\* | .73\*\* | .74\*\* |  |
| 12.expedite6 | -.48 | -.15 | -.44 | -.50 | -.46 | .53 | -.23 | .08 | 1.00\*\* | .99\*\* | .66\* |

*Note.* Items 1 to 6, labeled "defer," reflect the delay of an immediate reward, while items 7 to 12, labeled "expedite," pertain to the acceleration of a future reward. Asterisks denote statistical significance: \* *p* < .05, \*\* *p* < .01.

**Table 3**

*Goodness-of-fit Indicators and Likelihood Ratio Test for Delay Discounting Models (n = 497)*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Model | *χ2* | *df* | *χ2diff* | *dfdiff* | CFI | RMSEA | SRMR |
| Single Factor | 1589.43\*\*\* | 54 |  |  | .72 | .24 | .11 |
| Two Factor | 408.79\*\*\* | 53 | 1180.6\*\*\* | 1 | .94 | .12 | .04 |
| Four Factor | 263.24\*\*\* | 48 | 145.55\*\*\* | 5 | .96 | .10 | .04 |

*\*\* p* < .01. \*\*\* *p* < .001.

**Figure 1**

*One Factor CFA Model of Delay Discounting*

**A diagram of a computer algorithm

Description automatically generated**

*Note.* The factor *dsc* represents the sole factor “discount”. Items starting with "df" correspond to defer-related items, while items beginning with "ex" correspond to expedite-related items. Arrows pointing from factors to items indicate factor loadings, while arrows pointing to items from the bottom denote error variances. Sample size (n) = 497.

**Figure 2**

*Two-Factor CFA Model of Delay Discounting*

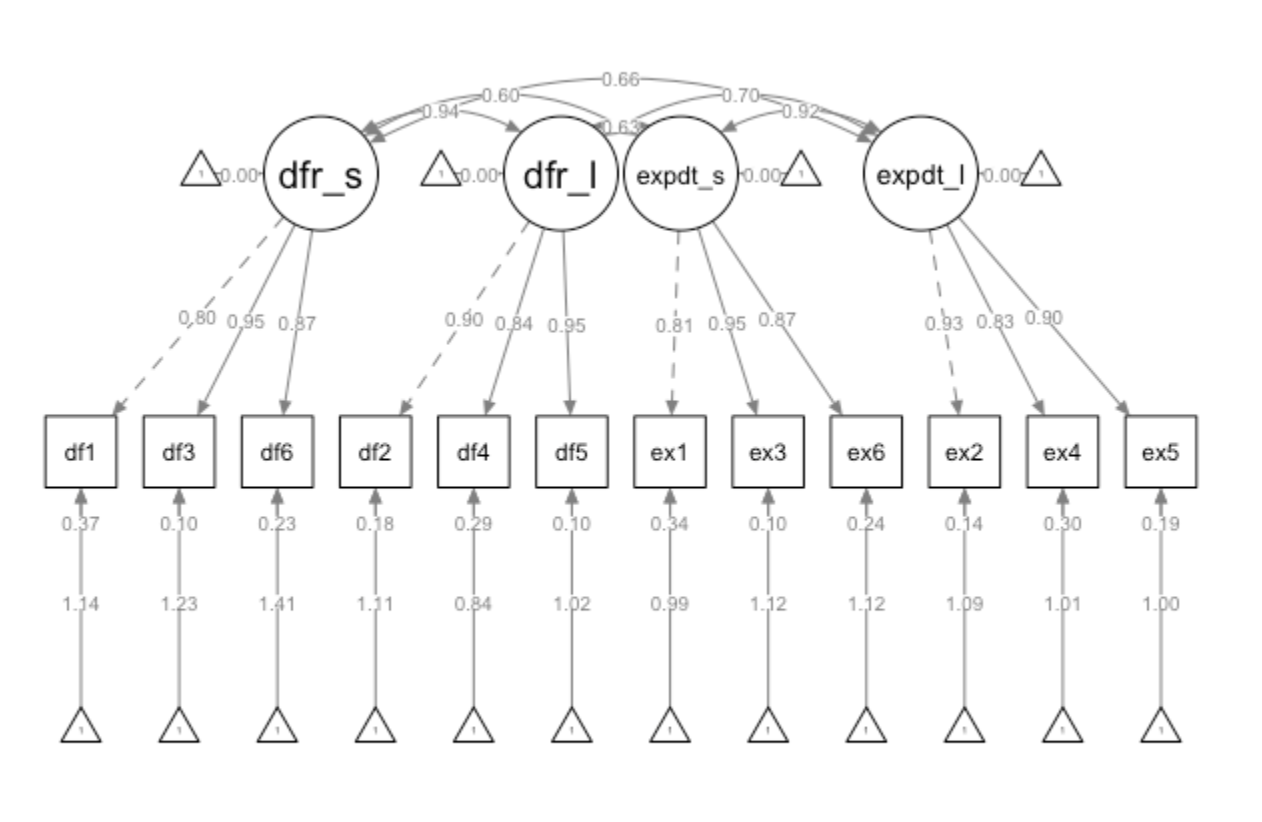
**A diagram of a algorithm

Description automatically generated**

*Note. dfr* refers to the first factor “defer”, and *exp* refers to the second factor “expedite.” Items 1 to 6, labeled "defer," reflect the delay of an immediate reward, while items 7 to 12, labeled "expedite," pertain to the acceleration of a future reward. Arrows pointing from factors to items indicate factor loadings, while arrows pointing to items from the bottom denote error variances. Sample size (n) = 497.

**Figure 3**

*Four-Factor CFA Model of Delay Discounting*



***Note*.** dfr\_s represents the first factor, "defer\_small," and dfr\_l represents the second factor, "defer\_large." Similarly, exp\_s and exp\_l refers to the third factor, "expedite\_small" and fourth factor "expedite\_large," respectively. Items 1 through 6, labeled "defer," measure the tendency to delay immediate rewards, while Items 7 through 12, labeled "expedite," assess the inclination to accelerate future rewards. Arrows pointing from factors to items indicate factor loadings, while arrows pointing to items from the bottom denote error variances. Sample size (n) = 497.